

# The constant background bag model of the hadron

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**Abstract.** We have constructed the bag model having a central constant color field. The motion of the quark is studied in this bag and the Dirac equation is solved for it. The energy spectrum found has a branching due to the interaction of the quarks with the color background. It is pointed out that this model can be applied for taking into account, in the mass spectrum of the hadrons, the coupling of the constituent quarks with the gluon condensation as the interaction with the color background.

## 1 Introduction

There are a constituent quark model and some bag models describing hadron states by means of the dynamics and kinematics of the constituent quarks [1–4]. These models take into account the gluon and quark condensation existing in the QCD vacuum [5] as a medium in which the constituent quarks move and the interaction of these quarks with condensates are reduced acquiring additional mass by them. The vacuum average values of the chromofield components  $\langle \mathcal{E}^a \mathcal{E}^a \rangle$  and  $\langle \mathcal{H}^a \mathcal{H}^a \rangle$  of the gluon condensate in the QCD vacuum were estimated in [5–7]. These estimations are used for the study of vacuum energy and structure of the bag models, which are connected with the bag constant [8,9]. Here we aim to consider the gluon condensate as a color background field and take into account the coupling of the constituent quarks with it as an interaction with the external color field. Naturally, such a coupling will arise in the constituent quarks energy spectrum which is not consisting in only acquiring an additional mass term and will result in a splitting of this spectrum. In order to provide the rotational invariance we propose that spatial components of this background are equal and constant for simplicity. The background field thus defined will play the role of a central constant color force. In order to realize our proposals we shall construct a model like Bogolubov's bag model [3], but having a constant color field in the center. According to this model the colored constituent spinor particle moves in the constant central color field and its motion is limited by an infinite spherical well. The condition of an infinite spherical well is imposed in order to ensure the confinement property of the colored constituents. The Dirac equation should be solved for this model under this boundary condition on wave functions of the constituent

quark. For this aim we can use the constant non-abelian vector potentials found in [10] and the approach used for solving the Dirac equation in the field given by such a kind of potential [11]. This will give us the distribution of the constituent quarks inside the bag and their energy spectrum, which will contain a contribution of its color and spin interactions with the background field. Finally, the components of the external field can be identified with the ones of the gluon condensate, and the estimations for the color components of the gluon condensation  $\langle \mathcal{E}^a \mathcal{E}^a \rangle$  and  $\langle \mathcal{H}^a \mathcal{H}^a \rangle$  can be settled in the energy spectrum found for the quark. This will make this spectrum useful for a comparison with the spectra of the hadron states.

## 2 The Dirac equation

For a study of the problems in a constant non-abelian background field it is convenient to introduce constant non-commuting vector potentials<sup>1</sup>. Let us choose the vector potentials  $A_\mu^{(a)}$  in the framework of the  $SU_c(3)$  color symmetry group in the following way:

$$\begin{aligned} A_0^{(a)} &= \sqrt{\tau_1}, & A_j^{(a)} &= \sqrt{\tau} \delta_{ja} \quad (j = 1, 2, 3) \text{ for } a = 1, 2, 3, \\ A_\mu^{(a)} &= 0 \text{ for } a = 4, 5, 6, 7, 8, \end{aligned} \quad (1)$$

where  $\tau, \tau_1$  are constants and  $\delta_{ia}$  is the Kronecker symbol. The corresponding field strength tensor  $F_{\mu\nu}^{(a)} = g f^{abc} A_\mu^{(b)} A_\nu^{(c)}$  has the following chromoelectric ( $\mathcal{E}_j^{(a)}$ ) and chromomagnetic ( $\mathcal{H}_j^{(a)}$ ) components:

$$\begin{aligned} \mathcal{E}_x^{(1)} &= 0, & \mathcal{E}_x^{(2)} &= g\sqrt{\tau\tau_1}, & \mathcal{E}_x^{(3)} &= -g\sqrt{\tau\tau_1}; \\ \mathcal{H}_x^{(1)} &= g\tau, & \mathcal{H}_x^{(2)} &= 0, & \mathcal{H}_x^{(3)} &= 0, \end{aligned}$$

<sup>1</sup> See [11] and references therein.

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$$\begin{aligned}
\mathcal{E}_y^{(1)} &= -g\sqrt{\tau\tau_1}, & \mathcal{E}_y^{(2)} &= 0, & \mathcal{E}_y^{(3)} &= g\sqrt{\tau\tau_1}; \\
\mathcal{H}_y^{(1)} &= 0, & \mathcal{H}_y^{(2)} &= g\tau, & \mathcal{H}_y^{(3)} &= 0, \\
\mathcal{E}_z^{(1)} &= g\sqrt{\tau\tau_1}, & \mathcal{E}_z^{(2)} &= -g\sqrt{\tau\tau_1}, & \mathcal{E}_z^{(3)} &= 0; \\
\mathcal{H}_z^{(1)} &= 0, & \mathcal{H}_z^{(2)} &= 0, & \mathcal{H}_z^{(3)} &= g\tau.
\end{aligned} \tag{2}$$

Here  $g$  is the color interaction constant. All other color components of  $F_{\mu\nu}^{(a)}$  are zero for  $a = 4, 5, 6, 7, 8$ . From (2) we see that the field (1) has equal magnitude of spatial components  $\mathcal{E}_j^2 = \mathcal{E}_j^{(a)}\mathcal{E}_j^{(a)} = 2g^2\tau\tau_1$  and  $\mathcal{H}_j^2 = \mathcal{H}_j^{(a)}\mathcal{H}_j^{(a)} = g^2\tau^2$ , which are constant as well. So, in ordinary space the strength vectors of the chromomagnetic and chromoelectric fields are<sup>2</sup>:

$$\vec{\mathcal{H}} = \sqrt{3}g\tau\vec{n}, \quad \vec{\mathcal{E}} = g\sqrt{6\tau\tau_1}\vec{n}, \tag{3}$$

where  $\vec{n}$  is the unit radius vector in ordinary space.

The Dirac equation for a colored particle minimally coupled with the external color field (1) can be written as follows:

$$(\gamma^\mu P_\mu - M)\Psi = 0, \tag{4}$$

where  $P_\mu = p_\mu + gA_\mu = p_\mu + gA_\mu^{(a)}\frac{\lambda^a}{2}$ , and the  $\lambda^a$  are the Gell-Mann matrices describing the particle's color spin. Equation (4) can be written in terms of Majorana spinors  $\phi$  and  $\chi$ ,

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

in a form more suitable for us:

$$(\sigma^j P_j)^2 \psi = (P_0^2 - M^2) \psi, \tag{5}$$

where the Pauli matrices  $\sigma_i$  describe a particle's spin. Here and afterwards  $\psi$  means  $\phi$  or  $\chi$ . The two spin components of the Majorana spinors

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

transform under the fundamental representation of the color group  $SU_c(3)$ . That means that each spin component of the wave function  $\psi_{1,2}$  has three color components describing the color states of a particle:

$$\psi_{1,2} = \begin{pmatrix} \psi_{1,2}^{(1)} \\ \psi_{1,2}^{(2)} \\ \psi_{1,2}^{(3)} \end{pmatrix}.$$

Writing down the expressions of  $P_\mu$  and  $A_\mu^{(a)}$  in (5) we get an explicit form:

$$\left( \vec{p}^2 + M^2 + \frac{3g^2\tau}{4} + g\tau^{\frac{1}{2}}\lambda^a p^a - \frac{g^2\tau}{2}\sigma^a\lambda^a \right) \psi$$

<sup>2</sup> We assume the chromoelectric field has negative projections in color space.

$$= \left( E^2 - g\tau_1^{\frac{1}{2}}\lambda^a E + \frac{3g^2\tau_1}{4} \right) \psi, \tag{6}$$

where  $E$  is the energy of particle<sup>3</sup>. Equation (6) turns into the following system of differential equations for the color components  $\psi_{1,2}^{(a)}$ :

$$\begin{cases}
\left( A + g\tau^{\frac{1}{2}}\mathcal{P}_3 - \frac{1}{2}g^2\tau \right) \psi_1^{(1)} + g\tau^{\frac{1}{2}}(\mathcal{P}_1 - i\mathcal{P}_2) \psi_1^{(2)} = 0, \\
\left( A - g\tau^{\frac{1}{2}}\mathcal{P}_3 + \frac{1}{2}g^2\tau \right) \psi_1^{(2)} + g\tau^{\frac{1}{2}}(\mathcal{P}_1 + i\mathcal{P}_2) \psi_1^{(1)} \\
= g^2\tau\psi_2^{(1)}, \\
(\vec{p}^2 + M^2) \psi_1^{(3)} = E^2\psi_1^{(3)}, \\
\left( A + g\tau^{\frac{1}{2}}\mathcal{P}_3 + \frac{1}{2}g^2\tau \right) \psi_2^{(1)} + g\tau^{\frac{1}{2}}(\mathcal{P}_1 - i\mathcal{P}_2) \psi_2^{(2)} \\
= g^2\tau\psi_1^{(2)}, \\
\left( A - g\tau^{\frac{1}{2}}\mathcal{P}_3 - \frac{1}{2}g^2\tau \right) \psi_2^{(2)} + g\tau^{\frac{1}{2}}(\mathcal{P}_1 + i\mathcal{P}_2) \psi_2^{(1)} = 0, \\
(\vec{p}^2 + M^2) \psi_2^{(3)} = E^2\psi_2^{(3)},
\end{cases} \tag{7}$$

where the operators  $A$  and  $\mathcal{P}_j$  denote  $A = \vec{p}^2 + M^2 + \frac{3}{4}g^2(\tau - \tau_1) - E^2$ ,  $\mathcal{P}_j = p_j + \sqrt{\frac{\tau_1}{\tau}}E$ . The equations in the system (7) mix the different states  $\psi_{1,2}^{(a)}$  and we need separated equations for each of these states. From the system (7) we find that the equations for all states  $\psi_{1,2}^{(i)}$  ( $i = 1, 2$ ) have the same form:

$$\begin{aligned}
& \left[ \left( A - \frac{g^2\tau}{2} \right)^2 - g^2\tau\vec{\mathcal{P}}^2 \right] \\
& \times \left[ \left( A + \frac{g^2\tau}{2} \right)^2 - g^2\tau(\vec{\mathcal{P}}^2 + g^2\tau) \right] \psi_{1,2}^{(i)} = 0, \tag{8}
\end{aligned}$$

which possesses rotational invariance. Since the operators  $A$  and  $\mathcal{P}_i$  commute, the operator in first square bracket commutes with the second one. This allows us to divide (8) into two equations, both keeping rotational invariance:

$$\left[ \left( A - \frac{g^2\tau}{2} \right)^2 - g^2\tau\vec{\mathcal{P}}^2 \right] \psi_{1,2}^{(i)} = 0, \tag{9}$$

$$\left[ \left( A + \frac{g^2\tau}{2} \right)^2 - g^2\tau(\vec{\mathcal{P}}^2 + g^2\tau) \right] \psi_{1,2}^{(i)} = 0. \tag{10}$$

Equations (9) and (10) can be solved separately and the set of solutions (8) will consist of the solutions of (9) and (10). Let us consider (9). Acting on this equation by the operator  $\vec{\mathcal{P}}^2$  we get the same equation for the function  $\xi_{1,2}^{(i)} = \vec{\mathcal{P}}^2\psi_{1,2}^{(i)}$  as for  $\psi_{1,2}^{(i)}$ , i.e.,

$$\left[ \left( A - \frac{g^2\tau}{2} \right)^2 - g^2\tau\vec{\mathcal{P}}^2 \right] \xi_{1,2}^{(i)} = 0.$$

<sup>3</sup> Since the field (1) does not depend on time, the states are stationary.

This means that the functions  $\xi_{1,2}^{(i)}$  and  $\psi_{1,2}^{(i)}$  differ only by a constant multiplier  $k'^2$ :  $\xi_{1,2}^{(i)} = k'^2 \psi_{1,2}^{(i)}$  or

$$\vec{\mathcal{P}}^2 \psi_{1,2}^{(i)} = k'^2 \psi_{1,2}^{(i)}. \tag{11}$$

In other words, since the operator  $\vec{\mathcal{P}}^2$  commutes with the square bracket operator in (9), they have the same set of eigenfunctions  $\psi_{1,2}^{(i)}$ . The same claim is in order for the operator  $\vec{p}^2$ :

$$\vec{p}^2 \psi_{1,2}^{(i)} = -\nabla^2 \psi_{1,2}^{(i)} = k^2 \psi_{1,2}^{(i)}. \tag{12}$$

Thus, we can solve (12) instead of (9). Obviously, (12) keeps the rotational invariance property of the equivalent equation (9) and so is easily solved in a spherical coordinate system using the separation ansatz [12]

$$\psi_{1,2}^{(i)}(\vec{r}) = R(r) \cdot Y_l^m(\theta, \varphi). \tag{13}$$

Here  $r = \sqrt{x^2 + y^2 + z^2}$ ,  $l$  and  $m$  are the orbital angular momentum and chromomagnetic quantum numbers, and  $\theta, \varphi$  are the polar and azimuthal angles, respectively. The spherical functions  $Y_l^m(\theta, \varphi)$  are expressed by means of the Legendre polynomials  $P_l^{|m|}(\cos \theta)$ :

$$Y_l^m(\theta, \varphi) = \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}} P_l^{|m|}(\cos \theta) e^{im\varphi}, \tag{14}$$

and define the s, p, d, f, ... orbitals well-known in quantum mechanics. The equation for the radial part  $R(r)$  is as used in many quantum mechanical problems possessing rotational invariance [11, 12]:

$$\frac{d^2}{dr^2} R(r) + \frac{2}{r} \frac{d}{dr} R(r) + \left( k^2 - \frac{l(l+1)}{r^2} \right) R(r) = 0. \tag{15}$$

With the notation  $Q(r) = \sqrt{r} R(r)$  (15) turns into Bessel's equation for  $Q(r)$ :

$$Q''(r) + \frac{1}{r} Q'(r) + \left( k^2 - \frac{(l+\frac{1}{2})^2}{r^2} \right) Q(r) = 0. \tag{16}$$

The function  $R(r)$  must be finite on  $r \rightarrow 0$ . This means that for the solution of (15) we should choose the Bessel function of the first kind:

$$R_l(r) = \frac{C_l}{k\sqrt{r}} J_{l+1/2}(kr). \tag{17}$$

Thus, we conclude that the states obeying (9) are the s, p, d, f, ... orbitals corresponding to different values of the quantum numbers  $l$  and  $m$  and the motion of the constituent quarks in any color and spin state takes place on these orbitals, which are the same for all these states. In other words, the angle distribution of quarks inside the bag is the same for any color and spin state and the same,

for instance, as the ones of electrons in atoms<sup>4</sup>. The radial distribution of these quarks is the same as the ones of freely moving particles enclosed in a sphere [12]. Since the external field (1) does not depend on  $r$ , we have obtained the same expression for the solutions  $\psi_{1,2}^{(i)}(\vec{r})$  as for a freely moving particle enclosed in a sphere, differing only by the expression of the  $k^2$  constant.

### 3 The energy spectrum

Using (3) the operator  $\vec{\mathcal{P}}$  can be written in the following form:

$$\vec{\mathcal{P}} = \vec{p} + \frac{E}{\sqrt{6g\tau}} \vec{\mathcal{E}}.$$

Then the action of the  $\vec{\mathcal{P}}^2$  operator will be

$$\vec{\mathcal{P}}^2 \psi_{1,2}^{(i)} = \vec{p}^2 \psi_{1,2}^{(i)} + \frac{2E}{\sqrt{6g\tau}} \left| \vec{p} \psi_{1,2}^{(i)} \right| \left| \vec{\mathcal{E}} \right| \cos \alpha + \frac{\tau_1}{\tau} E^2 \psi_{1,2}^{(i)}, \tag{18}$$

where  $\left| \vec{p} \psi_{1,2}^{(i)} \right|$  means the directional derivative [12]

$$\left| \vec{\nabla} \Phi \right| = \sqrt{\left( \frac{\partial \Phi}{\partial x} \right)^2 + \left( \frac{\partial \Phi}{\partial y} \right)^2 + \left( \frac{\partial \Phi}{\partial z} \right)^2}.$$

Here  $\alpha$  is the angle between the momentum vector  $\vec{\nabla} \psi$  and the chromoelectric field vector  $\vec{\mathcal{E}}$ . Since the particle periodically moves on the orbitals the angle  $\alpha$  varies in symmetric limits. That means that the average value of  $\cos \alpha$  during one period is zero. Actually, the term which contains  $\cos \alpha$  in (18) is proportional to the work done by the chromoelectric field on the particle in its motion in this field. It is easily seen that the net work of this field during one period is zero, while its momentary value is not zero<sup>5</sup>. So, if we average (9) and (10) over the time during one period, the term proportional to  $\cos \alpha$  will drop out. We shall find the average value of the energy spectrum<sup>6</sup>. Therefore, the average value of (18) during one period is equal to

$$\begin{aligned} \vec{\mathcal{P}}^2 \psi_{1,2}^{(i)} &= -\nabla^2 \psi_{1,2}^{(i)} + \frac{\tau_1}{\tau} E^2 \psi_{1,2}^{(i)} = k^2 \psi_{1,2}^{(i)} + \frac{\tau_1}{\tau} E^2 \psi_{1,2}^{(i)} \\ &= k'^2 \psi_{1,2}^{(i)}. \end{aligned} \tag{19}$$

<sup>4</sup> Since constituents in this model are considered to be non-interacting with each other, for the two-particle case we have the angle distribution of quarks in mesons. This distribution will have the same shape as the ones in a two-electron atom. In this sense mesons are like the hydrogen atom, differing only by the radial distribution in the constant field approximation.

<sup>5</sup> We observe the same situation as in an electron's motion in the field of a nucleus.

<sup>6</sup> For "s" orbitals  $\cos \alpha = 0$ , since the momentum and chromofield vectors are perpendicular on every moment in time. So, the momentary value of energy coincide with its average value for these orbitals.

If we take (19) and (12) into account, we obtain from (9) the following equation for the constant  $k^2$ :

$$(k^2)^2 + 2k^2 \left( M^2 - E^2 + \frac{1}{4}g^2(\tau - 3\tau_1) - \frac{1}{2}g^2\tau \right) + \left( M^2 - E^2 + \frac{1}{4}g^2(\tau - 3\tau_1) \right)^2 - g^2\tau_1 E^2 = 0,$$

from which one finds the relation between the constant  $k^2$  and the energy spectrum  $E^2$ :

$$(k^2)_{1,2} = \left( \sqrt{E^2 \left( 1 + \frac{\tau_1}{\tau} \right) - M^2 + \frac{3}{4}g^2\tau_1 \pm \frac{1}{2}g\tau^{\frac{1}{2}}} \right)^2 - E^2 \frac{\tau_1}{\tau}. \quad (20)$$

Now we should impose the boundary condition meaning the confinement property of the constituent quark on its wave function. Since (12) is the Laplace equation we can impose the Dirichlet boundary condition  $\psi_{1,2}^{(i)}(r = r_0) = 0$  or  $R_l(r_0) = 0$ , which means that we enclose the particle motion by a sphere with a radius  $r_0$ . Here  $r_0$  agrees with half of the hadron size. This boundary condition establishes the following relation between the values of  $k$  and the zeros  $\alpha_l^{(N)}$  of the Bessel function  $J_{l+1/2}(kr)$ :

$$kr_0 = \alpha_l^{(N)}, \quad (21)$$

which means the quantization of the  $k$  values. According to the relation (20) the energy spectrum of particle is quantized as well. Plugging (21) in (20), we find the first two branches of the quantized energy levels of the spectrum:

$$\left( E_i^{(N)} \right)_{1,2}^2 = \left( \sqrt{\left( 1 + \frac{\tau}{\tau_1} \right) \left( \frac{\alpha_l^{(N)}}{r_0} \right)^2 + M^2 + \frac{1}{4}g^2(\tau - 2\tau_1) \mp \frac{1}{2}g\tau_1^{\frac{1}{2}}} \right)^2 - \frac{\tau}{\tau_1} \left( \frac{\alpha_l^{(N)}}{r_0} \right)^2. \quad (22)$$

In (22) and (21)  $N$  labels the sequence of zeros of the Bessel function  $N = 1, 2, 3, \dots$  and is called the radial quantum number [12]. So, the value of the angular momentum  $l$  determines the series of the energy spectrum of the particle and the radial quantum number  $N$  determines the energy levels in this series. Thus, the finiteness condition imposed on the motion of the particle, because of its confinement property, of course, leads to the quantization of the energy spectrum.

As is seen from (21) the constant  $k$  gets the same values for the various branches  $(k^2)_{1,2}$ :

$$k_l^{(N)} = \frac{\alpha_l^{(N)}}{r_0}.$$

The radius  $a$  of the turning point of the particle could be found from (15) using the maximum condition on the radial function  $R'(a) = 0$  and is equal to

$$a_l^{(N)} = r_0 \frac{\sqrt{l(l+1)}}{\alpha_l^{(N)}}.$$

These radii, as the energy spectrum and  $k$ , are quantized and are determined by the quantum numbers  $l$  and  $N$ ; they do not depend on the field intensities. So, there is no difference in the values of the radii with the case of motion in a pure chromomagnetic field [11].

We have solved (9) and found the corresponding two branches of the energy spectrum. In the same manner we can solve (10) too. For this equation the function  $\xi_{1,2}^{(i)}$  denotes  $\xi_{1,2}^{(i)} = (\vec{p}^2 + g^2\tau) \psi_{1,2}^{(i)}$  and the following equations are equivalent to (10):

$$(\vec{p}^2 + g^2\tau) \psi_{1,2}^{(i)} = K^2 \psi_{1,2}^{(i)}, \quad (\vec{p}^2 + g^2\tau) \psi_{1,2}^{(i)} = K'^2 \psi_{1,2}^{(i)}. \quad (23)$$

The relation between the expressions of  $k$  and  $K$  is obvious:  $K^2 = k^2 + g^2\tau$ ,  $K'^2 = k'^2 + g^2\tau$  and the relation between  $K$  and  $K'$  is analogous to (19):

$$K'^2 = K^2 + \frac{\tau_1}{\tau} E^2. \quad (24)$$

Taking (23) and (24) into account, we get from (10) an algebraic relation between  $k^2$  and  $E^2$ :

$$\left( k^2 + M^2 + \frac{3}{4}g^2(\tau - \tau_1) - E^2 + \frac{g^2\tau}{2} \right)^2 - g^2\tau \left( k^2 + \frac{\tau_1}{\tau} E^2 + g^2\tau \right) = 0. \quad (25)$$

Of course, the solution of (23) is (13) with (14) and (17). The Dirichlet boundary condition  $R_l(r_0) = 0$  applies to (10) too. In the result of the quantization (21), from (25) we find the other two branches of the energy spectrum:

$$\left( E_i^{(N)} \right)_{3,4}^2 = \left( \sqrt{\left( 1 + \frac{\tau}{\tau_1} \right) \left( \left( \frac{\alpha_l^{(N)}}{r_0} \right)^2 + g^2\tau \right) + M^2 + \frac{1}{4}g^2(\tau - 2\tau_1) \mp \frac{1}{2}g\tau_1^{\frac{1}{2}}} \right)^2 - \frac{\tau}{\tau_1} \left( \left( \frac{\alpha_l^{(N)}}{r_0} \right)^2 + g^2\tau \right). \quad (26)$$

As seen from (22) and (26), the energy spectrum of the quark contains the contribution of the quark's interaction

with the background field and this energy spectrum is quantized due to the finiteness of the motion of the quark. Another difference with the existing bag models [1–4] is the branching of the energy spectrum, which is the result of the color and spin interaction of the particle with the background field and is not determined by these quantum numbers. We have four color and spin states  $\psi_{1,2}^{(i)}$  of the quark and four energy branches  $(E_l^{(N)})_{1,2,3,4}$ , but we do not have a one-to-one correspondence of these states and the branches. All these states could get energy from any of these branches. This is the difference between the branching of the spectrum and the splitting levels. In fact, here we have obtained the branching of the energy spectrum instead of splitting levels. Such a branching takes place in a pure chromomagnetic field case [11, 14, 15] and is connected with the existence of a conserved operator, which contains the color spin  $\frac{\lambda_a}{2}$  and the spin  $\frac{\sigma_j}{2}$  operators. There is no need to introduce the color states in the bag models, which do not deal with the color interaction of the particle with the background, and so, this branching of the energy spectrum does not occur in those models. Note that the energy levels in these branches are determined only by the quantum numbers  $l$  and  $N$ , and so they are  $2l + 1$ -fold degenerate in the quantum number  $m$ , which occurs in the motion in any central field.

The states  $\psi_{1,2}^{(3)}$  correspond to the states of a colorless particle, and (12) for them ( $k^2 = E^2 - M^2$ ) has the solution (13) with (15) and (17) too. The energy spectrum of these states is

$$(E_l^{(N)})_{5,6}^2 = \left(\frac{\alpha_l^{(N)}}{r_0}\right)^2 + M^2. \quad (27)$$

As an application of this model we can propose that the central color field is the background field of gluon condensation in the QCD vacuum. Then we can use the estimation for vacuum average values of the chromoelectric and chromomagnetic field intensities of this condensation [6, 7]:

$$\begin{aligned} \langle 0 | g^2 \vec{\mathcal{E}}^a \vec{\mathcal{E}}^a | 0 \rangle &\simeq - (700 \text{ MeV})^4, \\ \langle 0 | g^2 \vec{\mathcal{H}}^a \vec{\mathcal{H}}^a | 0 \rangle &\simeq (700 \text{ MeV})^4. \end{aligned}$$

Identifying these estimates with the field intensities in our model  $g^2 \vec{\mathcal{E}}^2 = 6g^4 \tau \tau_1$  and  $g^2 \vec{\mathcal{H}}^2 = 3g^4 \tau^2$ , we see that the constant  $\tau_1$  should be taken equal to  $\tau_1 = -\frac{1}{2}\tau$ . This enables us to evaluate the  $g^2 \tau$  constant  $g^2 \tau \simeq \frac{1}{\sqrt{3}} (700 \text{ MeV})^2$ . Then the energy spectra (22) and (26) simplify as follows:

$$\begin{aligned} (E_l^{(N)})_{1,2}^2 &= 2 \left(\frac{\alpha_l^{(N)}}{r_0}\right)^2 \\ &- \left( \sqrt{\left(\frac{\alpha_l^{(N)}}{r_0}\right)^2 - \frac{1}{2}g^2 \tau - M^2} \mp \frac{1}{2\sqrt{2}}g\tau^{\frac{1}{2}} \right)^2, \\ (E_l^{(N)})_{3,4}^2 &= 2 \left(\frac{\alpha_l^{(N)}}{r_0}\right)^2 + 2g^2 \tau \end{aligned} \quad (28)$$

$$- \left( \sqrt{\left(\frac{\alpha_l^{(N)}}{r_0}\right)^2 + g^2 \tau} - \frac{1}{2}g^2 \tau - M^2 \mp \frac{1}{2\sqrt{2}}g\tau^{\frac{1}{2}} \right)^2.$$

In order to estimate the quark’s energy using the spectra (28) and (27) we can set in them  $r_0 = R \simeq 1,7 \text{ fm}$  [3] or another estimation for the hadron’s size. Since the constituent quarks in the framework of this model are considered to be non-interacting with each other, the energy of any two- or three-quark system will be the sum of the energy of the separate quarks. As an examination of this model, the total energy of the bag can be compared with the corresponding hadron state. It should be noted that the selection rule  $\Delta m = 0, \pm 1$  and  $\Delta l = \pm 1$  for quantum transitions between the energy levels, which exists in central field problems [12], occurs for this problem as well and is useful for a calculation of the energy emitted by the excited bag. There is not a preferred rule on the energy branches for this calculation. A comparison of the energies emitted by the bag and by the hadron could serve as a another verification of the role of the gluon condensate as a constant central color field.

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